

For
GATE – PSU

Mechanical Engineering

Industrial Engineering

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INDUSTRIAL ENGINEERING AND OPERATION RESEARCH

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1. QUALITY MANAGEMENT

QUALITY

- Quality is a relative term and usually used with the reference to the end use of the product.
- Quality is defined as the fitness for the use at the most economical level.
- Quality is the ability of the component to perform satisfactorily in an application for which it is intended by the user.
- Quality depends mainly on the customer's perception.
- A product must have the following abilities to perform satisfactorily:-
 1. Suitability
 2. Reliability
 3. Durability
 4. Workability
 5. Affordability
 6. Maintainability
 7. Aesthetic look
 8. Satisfaction to customer
 9. Economical
 10. Versatility

QUALITY OF DESIGN

- It is concerned with the tightness of specifications for manufacture of the product.
- A good quality of design must ensure consistence performance over its stipulated life span stated in terms of rated output, efficiency, overload capacity, continued operation for specified application.

FACTORS CONTROLLING QUALITY OF DESIGN

1. Type of customer in the market.
Market survey is helpful for study of optimum quality of design. It involves:
 - Consuming habit of people
 - The prices they are willing to pay
 - Choice of design of product which meets the needs of customers.

Quality of design also depends upon type of customers i.e. rich, middle or poor to provide the intended function with greatest overall economy.

2. Type of goods

For capital goods, decision is governed by considerations such as intended life, environmental conditions, reliability etc.

3. Profit consideration

Profit can be maximized by producing products in different grades to suit different type of customers.

4. Environmental conditions

A good quality product at one place may be worst at another place according to environmental conditions.

QUALITY OF CONFORMANCE

- It is concerned with the how well the manufactured product conforms to the quality of design.
- Factors governing the quality of conformance:
 - Adequate quality of raw materials
 - Proper selection of process and adequate process control
 - Well trained and experienced operators.
 - Proper shipment and storage of finished goods
- Higher quality of design costs more while higher quality to conformance costs less, by reducing the number of defective items produced.

QUALITY OF PERFORMANCE

- It is concerned with how well the manufactured product gives its performance. It depends upon:
 - Quality of design
 - Quality of conformance
- A best design but with poor conformance control can cause poor performance and conversely, best conformance control cannot make the product function properly, if the design is not right.

QUALITY CONTROL

- CONTROL can be defined as a process by means of which we observe the actual performance and compare it with some standards.
- If there is any deviation between observed performance and the actual performance, some corrective action must be taken.
- Quality control is the process through which we measure the actual quality performance and compare it with some standard and take some corrective action if there is any deviation.
- It is the systematic control of the various factors that affect the quality of the product.

STEPS IN QUALITY CONTROL

1. Formulate the quality policy.
2. Work out details of product requirements, set the standards as per market survey.
3. Select inspection plan and set up procedure for for checking.
4. Detect deviations from set standard specifications.
5. Take corrective action and make necessary changes to achieve standards.
6. Decide on how the defective parts are disposed off or rework.
7. Coordination of quality problems.
8. Developing quality consciousness in the organization.

OBJECTIVES OF QUALITY CONTROL

- To improve company's income by making the product more acceptable to customers.
- To reduce company's cost by reducing losses due to defective items.
- To achieve interchangeability of manufacture in large scale production.
- To produce optimum quality at minimum price.
- To ensure satisfaction of customers with products or services of high quality level.

QUALITY CHARACTERSTICS

Quality characteristics may be:

1. Technological e.g. length, diameter etc.
2. Psychological e.g. taste, odour etc.
3. Time oriented e.g. reliability, maintainability etc.
4. Contractual e.g. guarantee, safety etc.
5. Ethical e.g. Honesty, integrity etc.

Quality characteristics may also be classified as

1. Directly measurable e.g. weight, shear strength etc.
2. Non-measurable e.g. rejection due to flaws, cracks etc.

VALUE OF QUALITY

- Value of quality may be defined as the return direct or indirect gained by manufacturer due to mission of quality control.
- Value of quality is composed of :
 - Value inherent in the design, usually called grade.
 - Value inherent in the conformance to that design.

STATISTICAL QUALITY CONTROL

- A quality system performs inspection, testing and analysis to ensure that the quality of the product produced as per set standards..
- It is called statistical quality control when statistical techniques are employed to control, improve and maintain quality.
- Statistics is the collection, organization, analysis, interpretation and presentation of data.

S.Q.C. consists of three general activities:

1. Systematic collection and graphical recording of accurate data.
2. Analysing the data
3. Practical engineering or management action, if the information obtained indicates significant deviations from the specified limits

STATISTICAL TOOLS FOR QUALITY CONTROL

Following tools are used for exercising control, improvement of quality etc.

1. Frequency distribution

A tabulation of the number of times a given quality characteristic occur within the samples.

2. Control charts

It is a graphical representation of quality characteristics, which indicates whether the process is under control or not.

3. Acceptance sampling

It is the process of evaluating a portion of production a lot for the purpose of accepting or rejecting the lot on basis of conformance.

4. Analysis of data

It includes special methods such as analysis of tolerances and variances, analysis of engineering designs.

THE CONCEPT OF VARIATION

- Concept of variation states that no two items will be perfectly identical even if extreme care is taken to make them identical in some respect.
- Variation may be very small as expressed in micron.

TYPES OF VARIATIONS

- The variations in the similar products may be due to chance or due assignable causes.

These variations may be classified into broad categories as follow:

1. Variation within part itself.
2. Variation among parts produced during same period of time
3. Variation among parts produced at different periods of time.

REASONS FOR VARIATIONS

1. Tool wear
2. Machine vibrations
3. Loose bearings
4. Carelessness of operator
5. Change of working conditions
6. Poor maintenance
7. Measuring errors

Statistical data may be characterized as

- Variable data
- Attribute data

VARIABLE DATA

- When record is made of an actual measured quality characteristic, such as dimensions in mm, quality is said to be expressed in variables.

- It is also known as continuous data i.e. Data on a characteristic that is measurable and can assume any value over some interval.
- Variability may be due to assignable causes of variation are likely to occur because of man, machine or material or it may be due to un-assignable causes.
- The objective of quality control is to minimize the un-assignable causes of variation and eliminate assignable causes of variation.

Examples of variables

- A dimension of a part measured
- Hardness in Rockwell units
- Temperature in degree centigrade

ATTRIBUTE DATA

- When a record shows only the no. of articles conforming and no. of articles failing to conform to any specified requirements, it is said to be recorded by attributes.
- This method simply notes the presence or absence of a quality characteristic in the parts examined.
- Attributes or discrete data is a data on characteristic that can assume certain distinct values.

Examples

- The no. of defective items found in a sample
- Surface finish of furniture

VARIATIONS DUE TO ASSIGNABLE CAUSES

- These possess greater magnitude as compared to those due to chance variations and can be easily traced.
- The Shewhart chart helps to separate out these assignable causes of quality variations.

These variations may be due to following factors:-

1. differences among the machines
2. differences among the workers
3. difference among the material
4. difference in each of these factors with time
5. Differences in their relationship to one another.

CHANCE VARIATION (RANDOM VARIATION)

- These variations are inevitable in any process or product.
- They are difficult to trace and difficult to control even under the best conditions of production.
- If after a random selection, observations are made under the same conditions and the distribution of observation follows a standard curve, it is assumed that the variations are due to chance causes and no assignable cause of error are present .
- The conditions which produce these variations are said to be under control.
- If variations in the data do not conform to the pattern that might reasonably produce by chance causes, then it is concluded that one or more assignable causes are at work. Such conditions are called out of control.

CONTROL CHART

- It is the graphical representation of the collected data. The data may pertain to measured quality characteristics of samples.
- Hence control chart
 - Specifies the state of statistical control
 - Helps attaining statistical control
 - Helps to judge whether statistical control has been attained.
- The control limits on the chart are so placed as to disclose the presence or absence of the assignable causes of quality variation.
- Control chart tells when to leave the process alone and prevents unnecessarily frequent adjustments that tend to increase the variability of the process.
- Many control charts are designed for different control situations.

The most commonly used control charts are:

1. Control charts for measurable quality characteristics.(\bar{X} and R chart, \bar{X} and σ chart) (for variables)
2. Control chart for fraction defective (P-chart) (for attributes)
3. Control chart for no. of defects per unit (C-defects) (for attributes)

CONTROL CHART FOR VARIABLES

- The control charts for variables are used for controlling fully automatic processes, where the operator is probably responsible for three or more machines.

- \bar{X} Chart shows the centering of the process i.e. shows the variation in the average of samples.
- R- Chart shows the uniformity of the process i.e. shows variations in the ranges of samples. It is a measure of spread.
- σ - Chart shows the variation of the process.
- These charts are used in combinations.

USE OF CONTROL CHARTS

- The control charts are used to determine whether a given process meet the existing specifications without a fundamental change in production process.
- To secure information to be used in establishing or changing the production process.
- To secure information when it is necessary to widen the tolerances.
- To secure information to be used in establishing or changing the inspection procedure or acceptance procedure or both.

RELATIONSHIPS BETWEEN \bar{X} , σ' AND VALUES OF \bar{X}

Consider 4 subgroups consisting of 5 items. Let $\bar{X}_1, \bar{X}_2, \bar{X}_3$ and \bar{X}_4 be the averages of first, second, third and fourth group respectively.

Then

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3 + \bar{X}_4}{4}$$

Where

$\bar{\bar{X}}$ = average of the averages

- When many random samples of any given size are taken from the universe, the average of samples \bar{X} will themselves form a frequency distribution.
- The spread of the frequency distribution depends upon the spread of universe as well as upon the sample size n.
- Larger the values of n, the less will be the spread of the \bar{X} values.

Statistical theory tells that in the long run,

$$I. \quad \bar{\bar{X}} = \bar{X}$$

i.e. averages of the average values will be the same as the average of universe.

$$\text{II. } \sigma_{\bar{X}} = \frac{\sigma'}{\sqrt{n}}$$

$\sigma_{\bar{X}}$ = Standard deviation of the expected frequency distribution of average

σ' = Standard deviation of universe

RELATIONSHIP BETWEEN $\bar{\sigma}$ AND σ'

Similar to the values of \bar{X} , values of σ' also differ from one subgroup to the next.

In the long run, the standard deviations of the samples of any size of the universe will follow a chance pattern.

$$C_2 = \frac{\bar{\sigma}}{\sigma'}$$

Where

$\bar{\sigma}$ = the average of standard deviations of samples of any given size.

C_2 = a factor and value can be seen from tables for different sample sizes.

RELATIONSHIP BETWEEN \bar{R} AND σ'

$$d_2 = \frac{\bar{R}}{\sigma'}$$

\bar{R} = Average range

d_2 = A factor and values can be read from table for various sample sizes.

CHOICE OF VARIABLE

- Variable for charts must be so chosen that it can be measured and expressed in numbers such as dimension, hardness no., tensile strength etc.

BASIS OF SUBGROUPING

We must be a very careful about selection of sub-group.

Following factors must be considered:

- Each sub-group must be homogenous as possible.

- There should be maximum opportunity for variation from one subgroup to another.
- Samples should not be taken at exactly equal intervals of time.

Size of subgroup

- Size of subgroup must be as small as possible to provide maximum homogeneity within subgroup.
- 4 or 5 is the most commonly accepted subgroup size. The distribution is normal for small sub-group. This is helpful in interpretation of control limits.
- Smaller subgroup provide for ease of computation of average.
- Sometimes, larger subgroup may be advantageous when it is desired to make the control chart sensitive to small variations.
- The large sample will cause limits of control charts to be closer to control line on the chart and becomes easy to detect even small variations.

Frequency of sampling

There are two possible ways of sampling:

- larger samples at less frequent intervals
 - if process remains in control
- Smaller sample at more frequent intervals.
 - If process is not under control
 - During the initial stages of process

CONTROL LIMITS

- For plotting control charts, ± 3 limits are selected and they are termed as control limits.
- They represent a band within which the dimensions of the components are expected to fall.
- 99.7 % of the samples fall within these limits. It means only 3 out of 1000 samples will fall outside these limits.

CALCULATION PROCEDURE

1. Calculate the average \bar{X} and range R for each subgroup.

$$\bar{X} = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$$

Range,

$$R = \text{highest value} - \text{smallest value}$$

2. Calculate the grand average $\bar{\bar{X}}$ and average range \bar{R}

$$\bar{\bar{X}} = \frac{\sum \bar{X}}{N}$$

$N = \text{no. of sub - groups}$

$$\bar{R} = \frac{\sum R}{N}$$

3. Calculate 3σ limits on control chart for \bar{X} chart.

$$\sigma' = \frac{\bar{R}}{d_2}$$

$$\sigma_{\bar{X}} = \frac{\sigma'}{\sqrt{n}}$$

$$3\sigma_{\bar{X}} = \frac{3}{\sqrt{n}} \frac{\bar{R}}{d_2}$$

$$\sigma_{\bar{X}} = A_2 \bar{R}$$

$$\text{Upper control limit for } \bar{X} = \bar{\bar{X}} + 3\sigma_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R}$$

$$\text{Lower control limit for } \bar{X} = \bar{\bar{X}} - 3\sigma_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R}$$

$$\bar{\sigma} = \frac{\sum \sigma}{N}$$

$$\sigma' = \frac{\bar{\sigma}}{C_2}$$

$$3\sigma_{\bar{X}} = \frac{3\bar{\sigma}}{C_2 \sqrt{n}} = A_1 \bar{\sigma}$$

$$\text{Upper control limit for } \bar{X} = \bar{\bar{X}} + 3\sigma_{\bar{X}} = \bar{\bar{X}} + A_1 \bar{\sigma}$$

lower control limit for $\bar{X} = \bar{\bar{X}} - 3\sigma_{\bar{X}} = \bar{\bar{X}} - A_1\bar{\sigma}$

$$3\sigma_{\bar{X}} = \frac{3\sigma'}{\sqrt{n}} = A\sigma'$$

Upper control limit for $\bar{X} = \bar{\bar{X}} + 3\sigma_{\bar{X}} = \bar{\bar{X}} + A\sigma'$

Upper control limit for $\bar{X} = \bar{\bar{X}} - 3\sigma_{\bar{X}} = \bar{\bar{X}} - A\sigma'$

CONTROL LIMITS FOR R-CHART

$$UCL_R = D_4\bar{R}$$

$$LCL_R = D_3\bar{R}$$

PROCESS CAPABILITY

- Minimum spread of a specific measurement variation which will include 99.7 % of the measurements from the given process.

$$\text{Process capability} = 6\sigma'$$

this is taken as the measure of the spread of the process

- This is also called natural tolerances.
- Process capability study is carried out to measure the ability of the process to meet the specified tolerances.
- A process capability analysis consists of:
 1. Measuring the process capability to find out whether the process is inherently capable of meeting the specified tolerances.
 2. Discovering why a process 'capable' is failing to meet specifications.

METHODS OF CALCULATING PROCESS CAPABILITY

1. Standard deviation method
2. The average range method
3. Single range method

ELEMENTS OF PROCESS CAPABILITY ANALYSIS

1. The specification tolerance
2. The determination of whether the process average is 'centered' mid-way between the tolerance limits.

3. Measurement of inherent variability of the process.
4. Measurement of actual variability over the period of time.
5. Causes of differences between inherent and actual variability.

PROCEDURE FOR ANALYSIS

1. Calculate the average \bar{X} and range R of each sample.
2. Calculate the grand average $\bar{\bar{X}}$. This measures the centering of the process.
3. Calculate control limits and plot \bar{X} and R charts.
This measures stability of the process.
4. Calculate the process capability $6\sigma' = 6\left(\frac{\bar{R}}{d_2}\right)$
This measures the piece to piece variability of the process.

POSSIBLE RELATIONSHIP OF A PROCESS IN CONTROL TO UPPER AND LOWER SPECIFICATION LIMITS

1. $(X_{max} - X_{min}) > 6\sigma'$

$X_{max} = \text{upper specification limit}$

$X_{min} = \text{lower specification limit}$

Spread of the process is considerably less than the difference of upper and lower limits.

With any position a, b and c, practically all the products manufactured will meet specifications as long as the process stays in control.

2. $(X_{max} - X_{min}) < 6\sigma'$

Spread is appreciably greater than the difference the specification limits.

In this situation, defective part will always be there.

So remedy will be

- Increase the tolerances
- Reduce the dispersion
- Suffer and sort out the defectives

3. $(X_{max} - X_{min}) = 6\sigma'$

Spread and the difference is approximately equal.

- It is necessary to take steps to retain centering of the process.
- It is advisable to increase tolerances.
- Reduce dispersion if it is economical.

CHARTS FOR ATTRIBUTES

p-CHART

- This is also known as fraction defective chart.
- The binomial distribution is the basis for the p-chart.

Control limits

$$UCL_p = p' + 3\sigma'_p = p' + 3\sqrt{\frac{p'(1-p')}{n}}$$

$$LCL_p = p' - 3\sigma'_p = p' - 3\sqrt{\frac{p'(1-p')}{n}}$$

- p-charts have a lower inspection and maintenance cost.
- It is helpful to discover the average proportion of defective articles submitted for inspection over a period of time.

np-CHART

- When subgroup size is variable, p-chart is used.
- If subgroup size is constant for actual no. of defectives, np-chart is used.

control limits

average fraction defectives,

$$\bar{p} = \frac{\sum np}{\sum n}$$

$$UCL_{np} = n\bar{p} + 3\sigma_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$LCL_{np} = n\bar{p} - 3\sigma_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

C-chart

- Based upon Poisson distribution.

- Poisson distribution is not asymmetrical distribution, the upper and lower do not corresponds to equal probabilities of a point on the control chart falling outside the limits.
- Area of opportunity for occurrence of defects should be fairly constant from period to period.
- Opportunities for defects are large while chances of a defect occurring in any one spot are small.

Control limits

$$\sigma_c = \sqrt{np'} = \sqrt{C'}$$

$$UCL_c = C' + 3\sqrt{C'}$$

$$LCL_c = C' - 3\sqrt{C'}$$

$$C' = \text{centre line}$$

u-chart

- When the sub-group size varies from sample to sample u-chart is used.
- Control limits on u-chart will also vary.

$$u' = \frac{C}{n} = \frac{\text{No. of defectives in a samle}}{\text{no. of units in a sample}}$$

$$UCL_u = u' + 3 \sqrt{\frac{u'}{n}}$$

$$LCL_u = u' - 3 \sqrt{\frac{u'}{n}}$$

$$u' = \text{center line}$$

ACCEPTANCE SAMPLING

- Acceptance sampling is the process of evaluating a portion of the product/material in a lot for purpose of accepting or rejecting a lot as either confirming or not conforming to the quality specifications.
- It can be carried out either by 100% inspection(all products are subjected to inspection) or sampling inspection (samples are selected from the lot and inspected).
- Destructive test must be carried out by sampling only.
- Less damage to product as only few items subjected to handling.

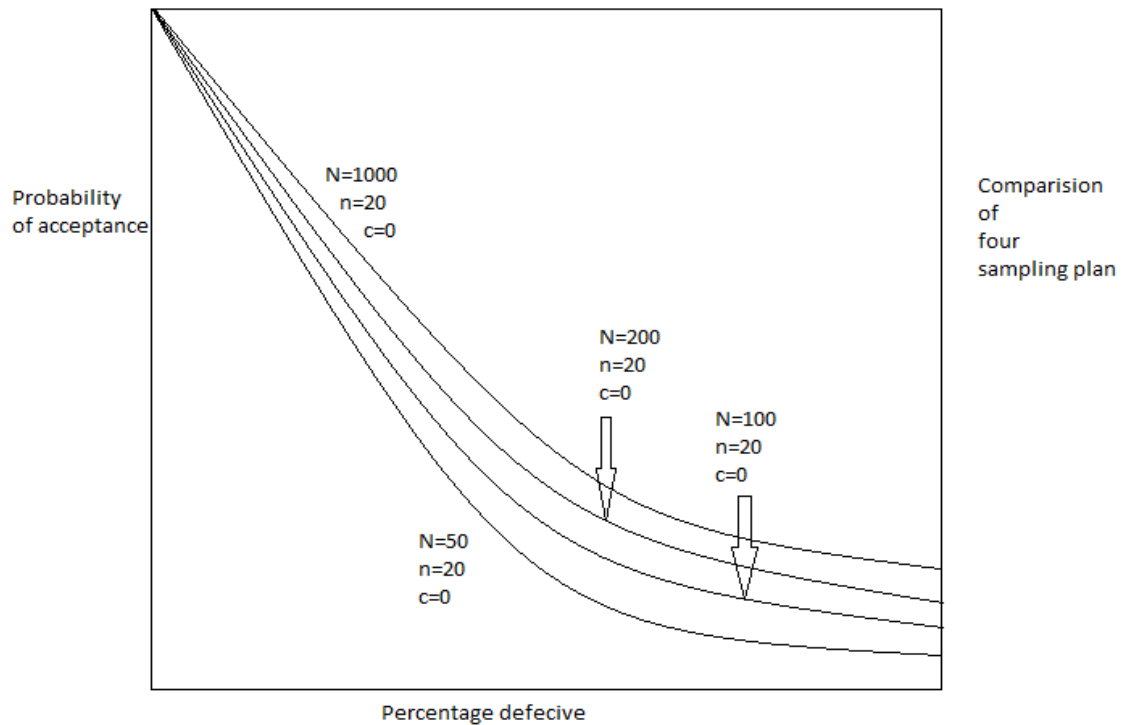
SAMPLING METHODS

1. Simple random sampling
 - Sample size 'n' is drawn from a lot size of 'N', in such a manner that while selecting an item, the chance for any item of being induced in sample is same.
2. Stratified sampling
 - Inspection lots are divided into sub-lots on the basis of homogeneity.
 - Draw sample items from all parts of such sub-lots of inspection lots.
 - Draw sample items blind.
 - More efficient than simple random sampling.
3. Systematic sampling
 - In this method, one item is chosen randomly from the lot and then items are selected at predetermined intervals.
4. Cluster sampling
 - if lot submitted for inspection consists of group of clusters of items.
 - it is advantageous to select a few cluster of items than examine all parts in the selected clusters.
5. Two-stage sampling
 - When the lot submitted for inspection consists of large number of items, then two stage sampling is used.
 - Initially, a desired no. of packages are selected at random and then at the second stage, required no. of items are chosen at random from selected primary units.

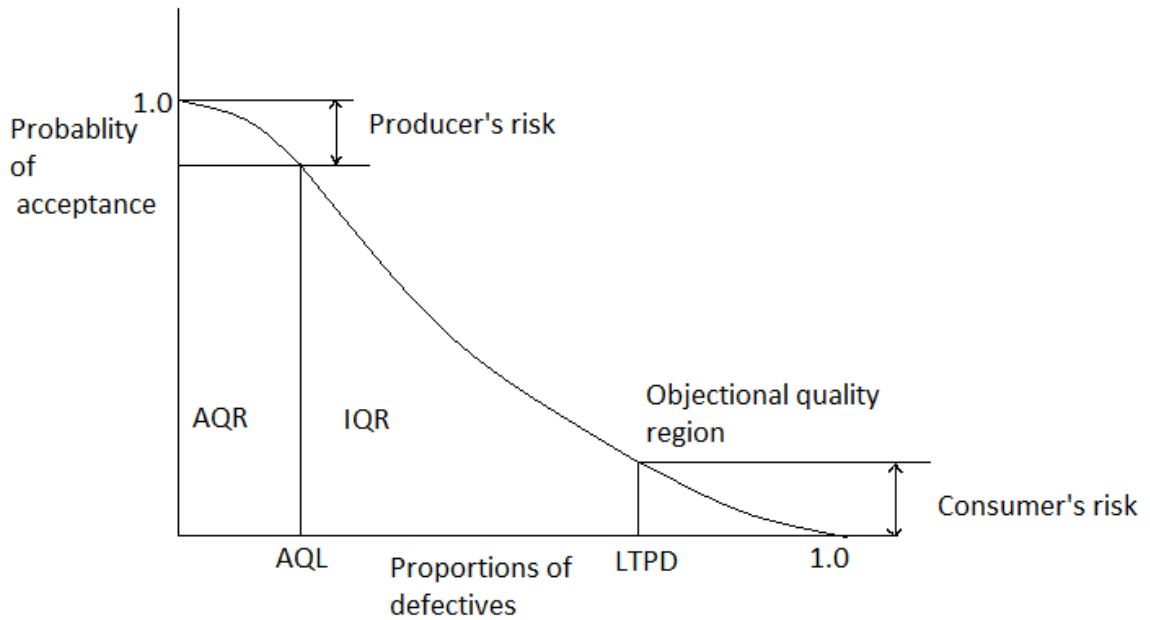
OPERATING CHARACTERISTICS

- Operating characteristics curve for an attribute sampling plan is a graph of fraction defective in a lot against the probability of acceptance.
Important parameters in a sampling plan
 - N = lot size from which samples are drawn

- N = sample size
- C = acceptance no.



- A sampling plan which will satisfy both consumer and the producer is not possible. Both of them will have to tolerate some risk.



Producer's risk

- It is the probability of rejecting a good lot which would have been accepted.
- The producer can decrease his risk by producing products at a better quality level than specified AQL.

Consumer's risk

- It is the probability of defective lot being accepted which otherwise would have been rejected.

ACCEPTABLE QUALITY LEVEL

- It represents the maximum proportion of defectives which the consumer finds definitely accepted.
- Probability of acceptance of AQL must be high.

REJECTABLE QUALITY LEVEL

- It is also called as Lot Tolerance Per cent Defective (LTPD).
- It represents the proportion of defectives which consumer finds definitely unacceptable.
- Probability of acceptance of RQL lot must be below.

INDIFFERENT QUALITY LEVEL

- A quality level where probability of acceptance of 50%.
- It lies between AQL and RQL.

AVERAGE OUTGOING QUALITY

- It represents the average % defective in the outgoing products after inspection, including all accepted and all rejected lots which have been 100 % inspected and defective replaced by non-defectives.

$$AOQ = p_a \cdot p' \left(\frac{N - n}{N} \right)$$